JMEPEG (2008) 17:482–488
DOI: 10.1007/s11665-007-9185-1

1059-9495/$19.00

Determination of Elastic Modulus in a Bimaterial Through a One-dimensional Laminated Model

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(Submitted April 25, 2007; in revised form October 19, 2007)

A model based on classical laminated plate theory reduced to one-dimension is proposed for analysis and data reduction of tensile testing of a bimaterial. The model is formulated in such a way that, through simple measurement of the bimaterial elastic response in a tensile test, it is possible to obtain the elastic modulus of one of the materials composing the bimaterial, if the dimensions and modulus of the other material are known. The sensitivity of the model to different material and geometric parameters is examined. The accuracy of the model is investigated comparing the model predictions to independent tensile testing and simple rule of mixtures. The proposed model can be used in cases where the properties of one of the materials in a bimaterial are difficult to obtain directly because of geometric constrains, like in the case thin films bonded to a thicker substrate. The model could also be extended to multilayers and/or bending loading cases.

Keywords bimaterial, CLPT, elastic modulus, mechanical properties

1. Introduction

Bimaterial structures, where two materials are bonded together, appear in numerous applications like sensors, thin films, coatings, and microelectronic industry. Analysis of the mechanical behavior of bimaterials traces back to the early work of Timoshenko in thermal bending of bimetal systems (used as thermostats) (Ref 1). Since then, several studies have addressed the stress distribution in bimetals due to uniform heating or cooling (Ref 2). Most of the existent work related to bimaterials has been oriented to delamination, fracture, and crack growth at the bimaterial interface (Ref 3-5). A prerequisite for all of these (and other) analyses is knowledge of the material properties of both constituents. In some applications, however, the material properties of one of the constituents are unknown and may be difficult to measure. The difficulty to measure such properties is accentuated in some applications like thin film technology, where the film properties may be thickness-dependent (Ref 6, 7). This complexity has led to development of several methods used for determination of mechanical properties of thin films bonded to a thicker substrate (a kind of bimaterial structure). Traditional methods used to measure the elastic modulus of thin films bonded to a substrate are often thermo-mechanical, i.e., require heating of the bimaterial, measurement of the curvature, and a data reduction method based on thermal bending (Ref 8-10). These methods require dedicated optical instrumentation to detect small curvature variations in the bimaterial and introduce additional thermal variables into the problem. With the development of scanning probe microscopes, a method that has been receiving increased attention for determination of mechanical properties of thin film-substrate systems is nano-indentation (Ref 11, 12). This technique has also some drawbacks and limitations mainly because of the substrate influence on the determination of the film properties (Ref 13). In this work, a one-dimensional (1D) tension model of a bimaterial structure is developed in a convenient way to obtain the elastic modulus of one of the materials in the bimaterial. The analysis could be readily extended to multilayers and/or bending loading cases. Assuming that the geometric parameters and the elastic modulus of one of the constituent materials are known, inversion of the problem allows determination of the unknown elastic modulus, requiring only measurement of the bimaterial elastic response. Using this approach the elastic modulus is obtained through direct tensile testing coupled with an adequate data reduction method, consistent with the conventional definition of elastic modulus. The sensitivity of the proposed model to the most important geometric and material parameters is investigated. The model is validated with independent tensile testing of two individual materials (Polyester and Polypropylene) conforming a bimaterial selected to this aim. The proposed model could be applied, for example, to calculate the film modulus in thin film/substrate structures.

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2. Bimaterial Model

2.1 Laminated Analysis

A perfectly bonded bimaterial subject to uniaxial tension is analyzed here based on classical laminated plate theory (CLPT), specialized to 1D. The constituent materials are labeled as “1” (bottom one) and “2” (top one). The bimaterial

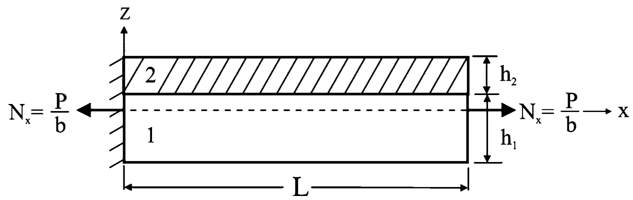


Fig. 1 Schematic representation of a bimaterial system showing some important parameters

is of length L, width b, and with thicknesses h_1 and h_2 for each material, see Fig. 1. The x-coordinate is the axial coordinate and the z-coordinate is the through-thickness coordinate, with $z = 0$ at the mid-plane $((h_1 + h_2)/2)$. A tension load, P, is applied uniformly along the x-axis. In this manner, the bimaterial system may be modeled using the first-order laminated theory as outlined by Whitney (Ref 14), here specialized to 1D. Within the linear elastic region, the stresses (σ) are proportional to strains (ε),

$$\sigma_x(i) = E_i \varepsilon_x(i) \quad \text{(Eq 1)}$$

Here, subscripts $i = 1, 2$ correspond to each material, and E is the elastic (Young's) modulus for isotropic materials, or the effective modulus for composite materials. According to the Kirchhoff hypothesis (Ref 14, 15), the axial displacement u of a point at (x, z) may be calculated using the mid-plane axial displacement u^0 and the rotation of the cross section $\psi(x)$, this is,

$$u = u^0(x) + z\psi \quad \text{(Eq 2)}$$

where $u^0 = u(x, z = 0)$, and ψ is the rotation of a cross section at x, originally plane and perpendicular to the specimen axis. The corresponding variation of strain through the thickness is given by,

$$\varepsilon_x = \varepsilon_x^0 + z\kappa_x \quad \text{(Eq 3)}$$

where ε_x^0 is the mid-plane strain, and κ_x is the mid-plane curvature given by,

$$\kappa_x = \frac{d\psi}{dx} \quad \text{(Eq 4)}$$

In general, the force and moment resultants, N_x and M_x, are defined as (Ref 14),

$$N_x = \int_{-h/2}^{h/2} \sigma_x \, dz \quad \text{(Eq 5a)}$$

$$M_x = \int_{-h/2}^{h/2} \sigma_x z \, dz \quad \text{(Eq 5b)}$$

Substitution of Eq 1 and 3 into Eq 5 yields, after integration and simplification,

$$N_x = A\varepsilon_x^0 + B\kappa_X \quad \text{(Eq 6a)}$$

$$M_x = B\varepsilon_x^0 + D\kappa_X \quad \text{(Eq 6b)}$$

where A, B, and D are, respectively, the 1D extensional, coupling and bending stiffnesses, given by,

$$A = E_1 h_1 + E_2 h_2 \quad \text{(Eq 7a)}$$

$$B = h_1 h_2 (E_2 - E_1)/2 \quad \text{(Eq 7b)}$$

$$D = \frac{1}{12}\left[E_1(h_1^3 + 3h_1 h_2^2) + E_2(h_2^3 + 3h_1^2 h_2)\right] \quad \text{(Eq 7c)}$$

It is assumed that both materials have a moderate interlaminar shear modulus and a large (>10) length-to-thickness ratio. Thus, shear deformation is expected to be minor and can be neglected.

For the case of tensile loading examined herein, since the only applied load is axial ($N_x = P/b, M_x = 0$, viz. Fig. 1), Eq 6b with $M_x = 0$ yield,

$$\kappa_x = \frac{-B}{D}\varepsilon_x^0 \quad \text{(Eq 8a)}$$

Substituting Eq 8a into Eq 6a yields the mid-plane axial strain ε_x^0,

$$\varepsilon_x^0 = \frac{N_x}{\left(A - \frac{B^2}{D}\right)} \quad \text{(Eq 8b)}$$

Since $\varepsilon_x^0 = \partial u^0(x)/\partial x$, the mid-plane axial deformation can be readily found integrating Eq 8b, resulting in,

$$u^0(x) = \frac{P}{b}\frac{D}{(AD - B^2)}x + C_0 \quad \text{(Eq 9)}$$

The boundary condition $u^0(x = 0) = 0$, which simulates the test setup (see Fig. 1), yields $C_0 = 0$. In this case, since only uniform tensile load (or strain) is applied to the specimen any cross section of the bimaterial displaces uniformly and $u(x, z) = u(x, z = 0) = u^0(x)$. Thus, the axial displacement of the examined specimen may be written as,

$$u(x) = \frac{P}{b}\frac{D}{(AD - B^2)}x \quad \text{(Eq 10)}$$

where A, B, and D, are defined in Eq 7.

Naming $u(x = L) = \delta$, the maximum axial displacement is given by,

$$u(x = L) = \delta = \frac{PL}{b}\frac{D}{(AD - B^2)} \quad \text{(Eq 11)}$$

It is worthwhile to notice that when $E_1 = E_2$ the coupling stiffness element vanishes ($B = 0$) and the conventional strength of materials formula $\delta = PL/Ebh (E = E_1 = E_2, h = h_1 + h_2)$ is recovered.

For convenience, Eq 11 may be written as,

$$\frac{PL}{b} = \frac{(AD - B^2)}{D}\delta = m\delta \quad \text{(Eq 12a)}$$

with

$$m = \frac{AD - B^2}{D} \quad \text{(Eq 12b)}$$

Equation 12a defines a parameter m as the initial slope of the *PL/b* vs. δ curve, see Fig. 2. Thus, m encompasses the stiffness of the bimaterial ($m = E_{\mathrm{Bim}}(h_1 + h_2)$, where E_{Bim} is the modulus of the bimaterial) and may be obtained plotting the load-displacement data of an actual tensile test in the format presented schematically in Fig. 2. The objective here is to use this bimaterial elastic response (m) to extract the unknown

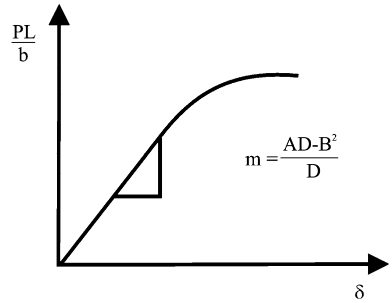


Fig. 2 Schematic representation of a *PL/b* vs. δ plot for a bimaterial system. The slope of the curve corresponds to the bimaterial elastic response *m* (Eq 12)

elastic modulus of one of the materials, say E_2, assuming that E_1 and all the geometrical parameters (L, b, h_1, h_2) are known. To simplify the calculations, let us define a new variable $\xi = E_2 h_2$ (with E_2 as unknown) and the following constants,

$$\varphi_1 = E_1 h_1 \quad \text{(Eq 13a)}$$

$$\varphi_2 = h_1/2 \quad \text{(Eq 13b)}$$

$$\varphi_3 = E_1 h_2 \quad \text{(Eq 13c)}$$

$$\varphi_4 = E_1\left(h_1^3 + 3h_2^2 h_1\right)/12 \quad \text{(Eq 13d)}$$

$$\varphi_5 = \left(h_2^2 + 3h_1^2\right)/12 \quad \text{(Eq 13e)}$$

Notice that the parameters φ_1 through φ_5 depend only on E_1, h_1, and h_2, which are assumed to be known. With these definitions, the expressions for the stiffness elements *A*, *B*, and *D* in Eq 7 may be rewritten as,

$$A = \xi + \varphi_1 \quad \text{(Eq 14a)}$$

$$B = \varphi_2(\varphi_3 - \xi) \quad \text{(Eq 14b)}$$

$$D = \varphi_4 + \varphi_5 \xi \quad \text{(Eq 14c)}$$

Substituting Eq 14a-c into Eq 12b and performing algebraic manipulations results in the following quadratic equation in the variable $\xi (= E_2 h_2)$,

$$\varphi_6 \xi^2 + \varphi_7 \xi + \varphi_8 = 0 \quad \text{(Eq 15)}$$

where

$$\varphi_6 = \varphi_5 - \varphi_2^2 = h_2^2/12 \quad \text{(Eq 16a)}$$

$$\varphi_7 = \varphi_4 + \varphi_1\varphi_5 + 2\varphi_3\varphi_2^2 - m\varphi_5 \quad \text{(Eq 16b)}$$

$$\varphi_8 = \varphi_1\varphi_4 - \varphi_2^2\varphi_3^2 - m\varphi_4 \quad \text{(Eq 16c)}$$

Thus, if L, b, h_1, h_2, and E_1 are known, and the bimaterial elastic response *m* is measured in a tensile test, φ_1 through φ_8 are readily obtained from Eq 13a-e and 16a-c and the unknown elastic modulus E_2 can be found solving Eq 15. The solution of Eq 15 yields two roots, a positive one and a negative one, where only the positive one has physical meaning.

For bimaterial systems where one of the constituent materials is much thinner than the other $(h_2 \ll h_1)$, such as in the case of thin film/substrate systems, φ_6 is negligible compared to φ_7 and φ_8 and consequently Eq 15 may be approximated as,

Table 1 Baseline geometric and material parameters used for the parametric analysis

Bimaterial system	*h*, mm	*h*, μm	*E*, MPa	*m*, N/mm
Macroscopic (PR/PP)	0.74	730	454.5	1013.5
Microscopic (Au/PP)	0.55	0.20	650	394.2

$$\xi \approx \frac{-\varphi_8}{\varphi_7} \quad \text{(Eq 17)}$$

It is important to point out that the approach presented here for determination of elastic modulus of one of the constituent materials in a bimaterial depends solely on the measurement of the elastic response of the bimaterial during a conventional tension test (assuming that the geometrical parameters and modulus of one of the materials are known). Also, substituting $N_x = 0$ and the adequate bending moment M_x into Eq 6a and 6b, similar equations could be derived for a bending loading case.

2.2 Parametric Analysis

A parametric analysis was conducted in order to examine the influence of the most important geometric and material properties of the bimaterial on the elastic modulus predicted by the model. In a preliminary study (not discussed herein), the parameters h_1, h_2, E_1, and *m* were identified as the most influential on the predicted modulus E_2; thus, our attention will be focused on these parameters. Two significantly different cases were studied: a so called "macroscopic system" composed of two materials with (nearly) equal thickness in the order of sub-millimeters, and a so called "microscopic system," composed of a thin film/substrate system with film thickness in the order of sub-microns and substrate thickness of sub-millimeters. Table 1 presents the baseline dimensions and moduli used for both bimaterial systems investigated. The baseline properties listed for the macroscopic system correspond to a bimaterial composed of Polyester/Polypropylene (PR/PP), to be discussed later in connection with the model validation. The properties listed for the microscopic system correspond to a gold thin film onto a polypropylene substrate (Au/PP). Both systems consider a span length (*L*) of 20 mm and a specimen width (*b*) of 5 mm. Figure 3 presents the predicted elastic modulus of material 2, E_2, for the macroscopic system, as a function of h_1, h_2, E_1, and *m*. To ease our discussion, we will refer to material 1 as the "substrate" with known modulus, and material 2 as the "adherent" with unknown modulus $(E_2 > E_1)$. First, the elastic bimaterial response (*m*), the thickness of the adherent (h_2), and the substrate modulus (E_1) were kept constant while the thickness of the substrate (h_1) was increased from 0.6 to 0.8 mm. Notice that keeping *m* constant implies that E_2 will change with the examined parameter (h_1, h_2, or E_1) to enforce the constant value of this bimaterial response. The later statement can be rationalized as a simple "rule of mixtures" where the effective (composite) property is held constant. The adherent modulus (E_2) predicted in this manner is shown in Fig. 3(a). It is observed that, if the bimaterial elastic response is kept constant, E_2 has to decrease with increased substrate thickness, although the decreasing rate is very low. If the thickness of the adherent h_2 is increased while the rest of the parameters are kept constant, a faster decrease in the adherent modulus is observed,

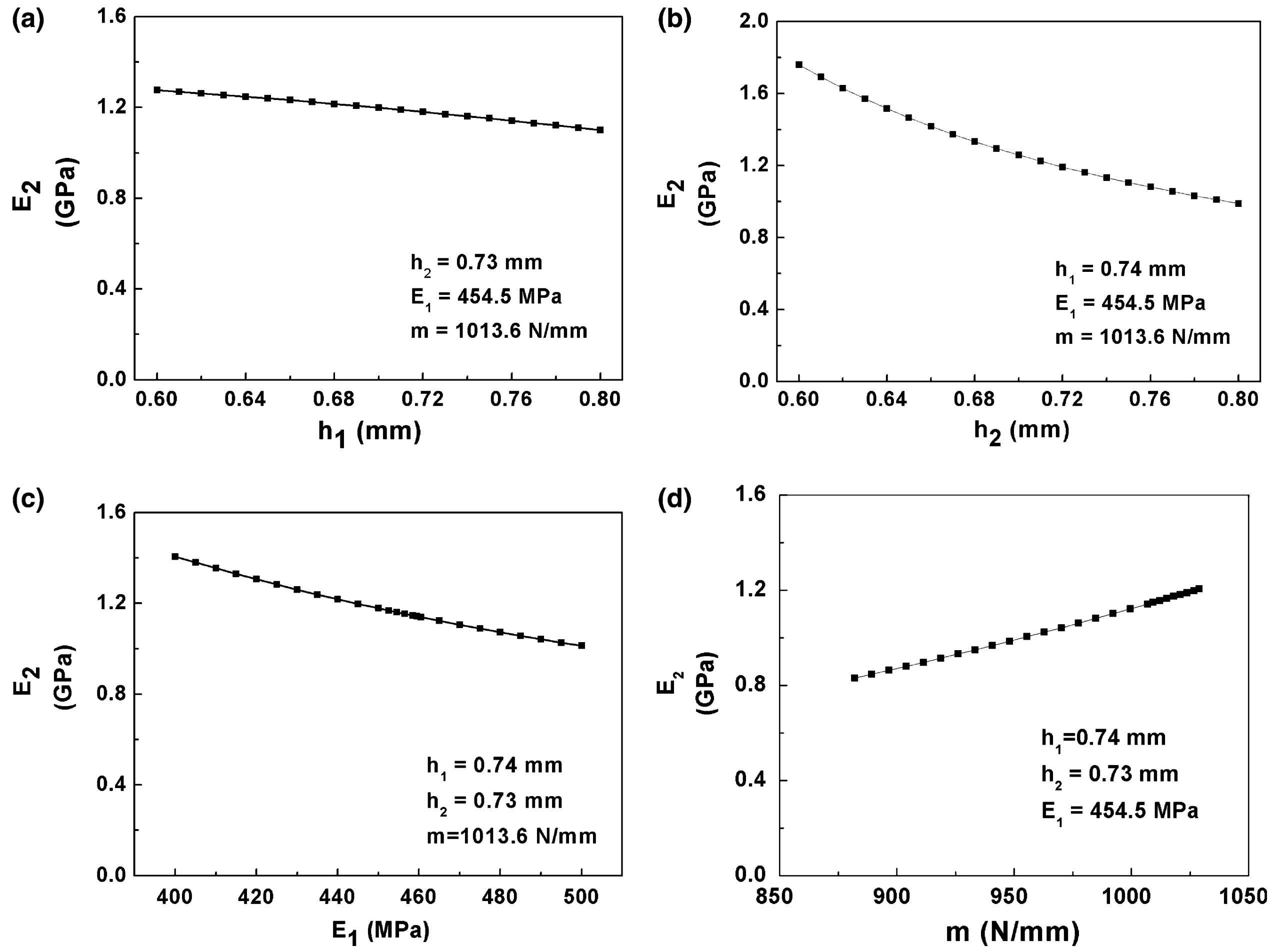


Fig. 3 Elastic modulus E_2 (adherent) predicted from Eq 15 for the macroscopic bimaterial system outlined in Table 1. (a) E_2 vs. h_1, (b) E_2 vs. h_2, (c) E_2 vs. E_1, and (d) E_2 vs. m

Fig. 3(b). A similar decreasing trend is observed when the substrate modulus is increased, Fig. 3(c). In such a case, for the bimaterial elastic response m to be constant, an increase in E_1 requires a decrease in E_2. A different trend is observed when the bimaterial elastic response m is increased while the rest of the parameters are held constant. For such a case, any increase in the bimaterial stiffness necessarily comes from an increase in the modulus of the adherent, cf. Fig. 3(d).

Analogous plots to those in Fig. 3 are presented in Fig. 4 for the microscopic (thin film/substrate) system, Table 1. Here, the substrate represents material #1 with known properties and the thin film represents material #2 with unknown elastic modulus E_2, see Fig. 1. The tendencies of the predicted film modulus, E_2, shown in Fig. 4 for the thin film/substrate system are similar to those of the adherent modulus in the macroscopic system, Fig. 3. The predicted modulus decreases with increased h_1, h_2, and E_1 (Fig. 4a-c), and increases with increased bimaterial elastic response m (Fig. 4d). However, for the thin film/substrate system (Fig. 4a-d), E_2 varies quite substantially with variations of the parameters studied (h_1, h_2, E_1, m), as opposed to the relative minor changes in E_2 observed for the macroscopic system (Fig. 3a-d). The increased sensitivity in the film/substrate system is due to the small thickness of the film compared to substrate thickness (substrate-to-film thickness ratio of the order of 3000). The results presented in Fig. 3 and 4 can further assist in the examination of the sensitivity of the model and in the determination of the key parameters of a bimaterial system. In Fig. 3(a), it is observed that the variation of E_2 with small changes in h_1 is almost negligible. More significant variations in E_2 are observed when h_2, E_1, and m are varied, Fig. 3(b-d). Thus, the most sensitive parameters for a macroscopic bimaterial system seem to be m, h_2, and E_1, although the changes produced in E_2 with small variations of these parameters are not really major. Small inaccuracies in h_1, h_2, E_1, and m may arise during measurement of the specimen dimensions and data reduction. For example, for the macroscopic system, the imprecision in E_2 that would occur if h_1 is measured as 0.75 mm instead of 0.74 mm is only 0.8% (1.15 MPa instead of 1.16 MPa). If h_2 is measured as 0.74 mm instead of 0.73 mm the inaccuracy in E_2 would be of 2.5% (1.13 MPa instead of 1.16 MPa). Similar percentages are observed for small variations in E_1 and m for the macroscopic system. Thus, the model is fairly robust to reasonable variations in material and geometric parameters of a bimaterial composed of materials of similar dimensions. For the film/substrate system, however, the sensitivity of the predicted elastic modulus to the same analyzed parameters (h_1, h_2, E_1, and m) greatly increases, viz. Fig. 4. For example, when the thickness of the substrate is changed from 0.50 to 0.51 mm a decrease in E_2 of 24.2% is observed, Fig. 4(a). Varying m from 440 to

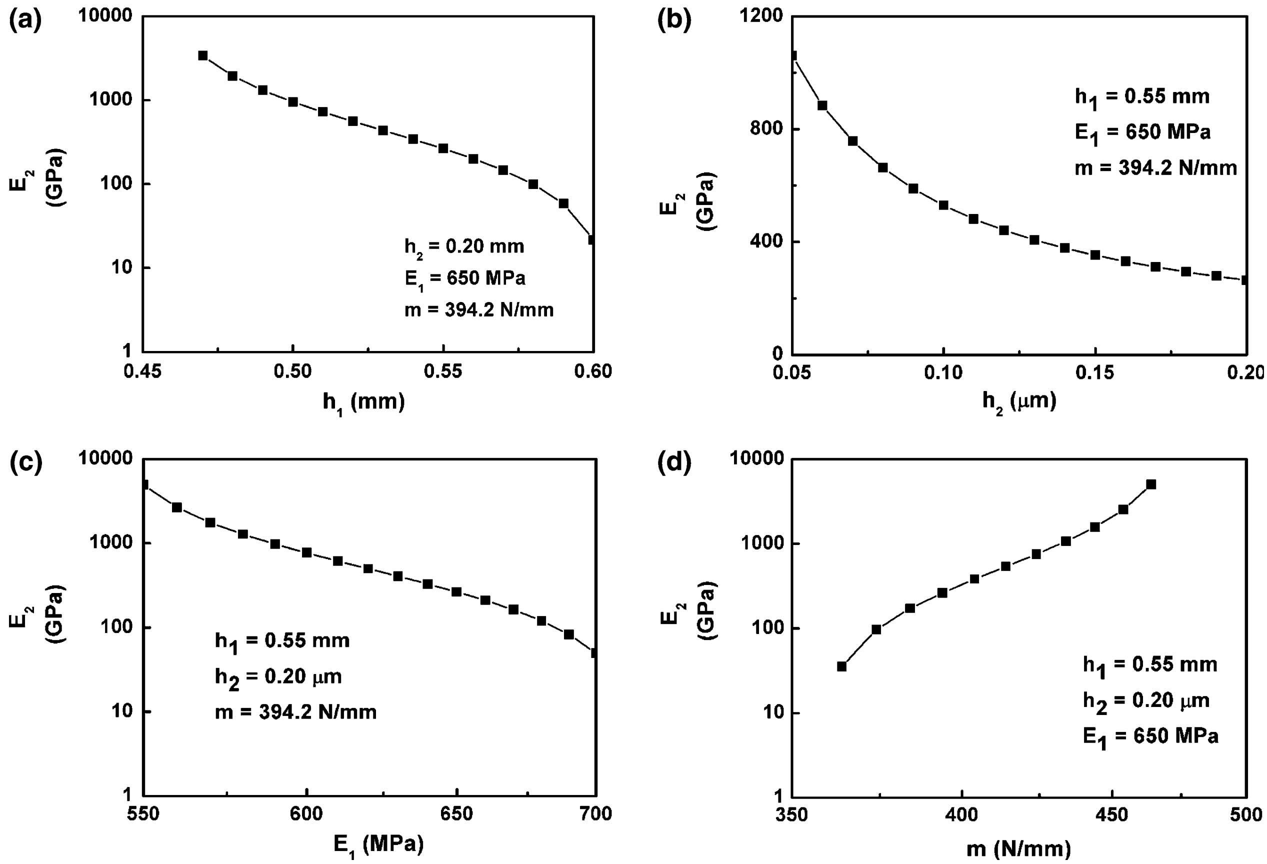


Fig. 4 Elastic modulus E_2 (film) predicted from Eq 15 for the thin film/substrate system outlined in Table 1. (a) E_2 vs. h_1, (b) E_2 vs. h_2, (c) E_2 vs. E_1, and (d) E_2 vs. m

450 N/mm yields an increase in E_2 of 37.7%, Fig. 4(d). Therefore, small inaccuracies in the specimen dimensions, substrate modulus, and bimaterial elastic response (which may originate from inaccurate measurements) may lead to imprecisions in the predicted film modulus. This is believed to be an inherent characteristic of the thin film/substrate system, caused by the very small film thickness compared to that of the substrate. In light of these findings, it is clear that accurate measurements of the geometric and material parameters of the bimaterial are imperative to warrant precision in the prediction of the unknown elastic modulus, especially for those bimaterials where one material is substantially thicker than the other.

3. Experimental Validation

In order to assess the model accuracy, a bimaterial was manufactured, tested, and the proposed approach was used to calculate the elastic modulus of one of the materials (assumed to be unknown) constituting the bimaterial. The elastic modulus calculated in this way was compared to the elastic modulus calculated from independent (conventional) tensile testing of the single material.

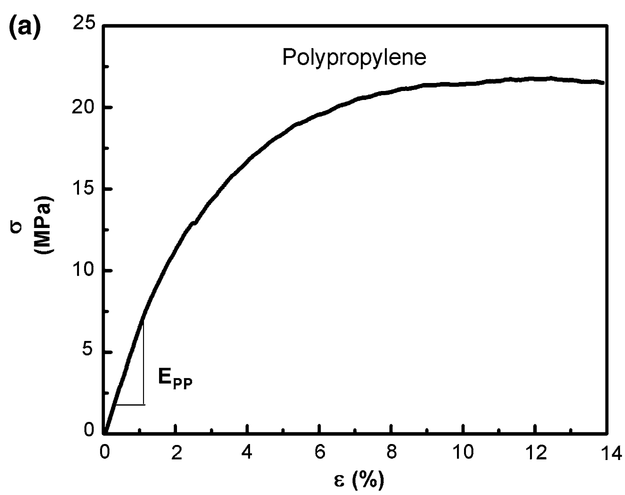
3.1 Specimen Manufacturing

Bimaterials of macroscopic (mm) dimensions were manufactured for testing and model validation. In order to avoid a potential third layer for gluing both materials together, polypropylene (PP) and a thermosetting polyester resin (PR) were employed. The PR can be directly bonded to PP avoiding the use of an additional adhesive layer. In this way, the PP acts as a template or "substrate" for the PR (adherent). A small (10×10 cm^2) PP plate was first manufactured by conventional compression molding. Pellets of PP were hot pressed inside a 0.74 mm thick aluminum mould to conform the plates. The PP plates were then marked and cut to obtain final specimens of nominal dimensions $30 \times 5 \times 0.74$ mm^3. For increased accuracy, the specimen's dimensions were measured with a digital caliper after cutting. To promote mechanical bonding between the PP and PR, several longitudinal thin and shallow grooves were made with a razor blade on the bonding surface of each of the PP substrates. A 10×10 cm^2 silicon mould with several rectangular grooves of $30 \times 5 \times 1.48$ mm^3 was manufactured to fabricate the bimaterials. The $30 \times 5 \times 0.74$ mm^3 PP specimens were fitted into the grooves of the silicon mould to be used as "substrates" for the PR deposition. The PR was then mixed with Methyl ethyl Ketone Peroxide (curing agent) at 0.8% in weight, stirred for 2 min and then poured into the grooves made on the silicon mould, previously filled with the solid PP substrates. The bimaterial specimens were then left for curing of the PR for 24 h at room temperature and then post-cured for 5 h at 70 °C. Additionally, neat PR specimens (without PP) were also fabricated pouring the PR resin into the ($30 \times 5 \times 0.74$ mm^3) grooves of a specially manufactured silicon mould, and following the same curing procedure as outlined above. The nominal thickness of the PP and PR specimens was 0.74 mm. After measurement, it was found that the dimensions of some specimens slightly differed from their nominal values, and the measured values were used for increased accuracy. For the PR/PP bimaterials, the thickness of the PP and bimaterial were actually measured and the thickness of the PR was inferred by difference.

3.2 Elastic Modulus from Tensile Testing and Analysis

Tensile tests of neat PP and PR specimens, and PR/PP bimaterials were conducted to assess the accuracy of the proposed model. The tests were conducted in a "minimat" testing machine with load cell of 200 N and a cross head speed of 0.05 mm/min. The specimens were nominally 30 mm long by 5 mm wide with a span length (distance between grips) of 20 mm. The nominal thickness was 0.74 mm for the PP and PR specimens, and 1.48 mm for the PR/PP bimaterials. Five replicates of each individual material (PP and PR) and the same number of PR/PP bimaterials were tested. The strain was calculated as the cross head displacement of the testing machine normalized by the 20 mm span length. The elastic modulus of the single PP and PR specimens was first calculated as the initial slope of the stress (σ)-strain (ε) curve, Fig. 5(a, b). The linear fit for modulus determination was conducted in a strain range of 0.1-0.7%. As it can be seen from these figures, polypropylene is a very ductile material with the ability to sustain very large strains, while the polyester resin is fragile with very limited deformation capacity. The linearity of the PP and PR curves in the initial portion of the σ-ε curve allows direct determination of the elastic modulus of each material, see Fig. 5(a, b). In this manner, the elastic moduli of the PP and PR were measured as 454.5 ± 10.7 MPa and 1258.3 ± 41.4 MPa, respectively, where the ±sign represents one standard deviation calculated from the five replicate testing. In general, the elastic modulus of thermoplastics and thermosetting resins depend on the processing and curing (for thermosettings) conditions.

Tensile testing of PR/PP bimaterials was conducted under similar experimental conditions to those presented above for single (neat) materials. The stress-strain curve of a typical PR/PP bimaterial is shown in Fig. 6, where the curves of the neat PP and PR have been superimposed. As observed in this figure, the presence of PP shifts the maximum stress and strain of the bimaterial to greater values with respect to the neat PR. At a strain of about 1.75% the PR in the bimaterial breaks (perpendicularly to the applied load) and the bimaterial curve drops. A tendency of the curve to recover the PP behavior is observed subsequently. Since the PR in the bimaterial broke at a strain of about 1.75%, tensile testing of the bimaterial was not extended to large deformations and the test was stopped at $\varepsilon \sim 4\%$. For determination of the bimaterial elastic response (m), the load-displacement curve generated from testing of the bimaterial was plotted in the format sketched in Fig. 2. In order to use the bimaterial model to extract the elastic model of one of the materials, the PP was assumed to act as a substrate with known elastic modulus (454.5 MPa) and the PR was assumed to be the material with "unknown" elastic modulus. With the dimensions of the PP and PR, elastic modulus of PP, and bimaterial elastic response (m) obtained from tensile testing of the bimaterial, the parameters φ_i ($i = 1 \ldots 8$) can be calculated from Eq 13a-e and 16a-c and the elastic modulus of the PR can be computed solving Eq 15. The elastic modulus of the polyester resin computed in this manner is shown in Table 2 for the five replicate specimens, along with the difference (in %) between the predicted modulus and the average modulus determined from direct tensile testing of the single PR



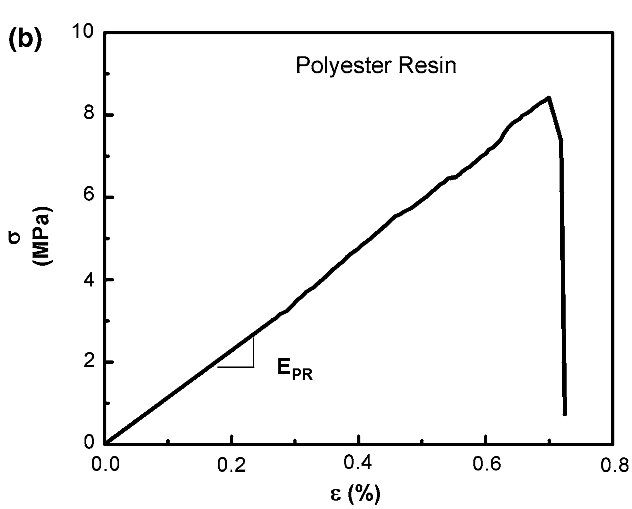


Fig. 5 Tensile stress-strain curves of neat PP (a) and PR (b)

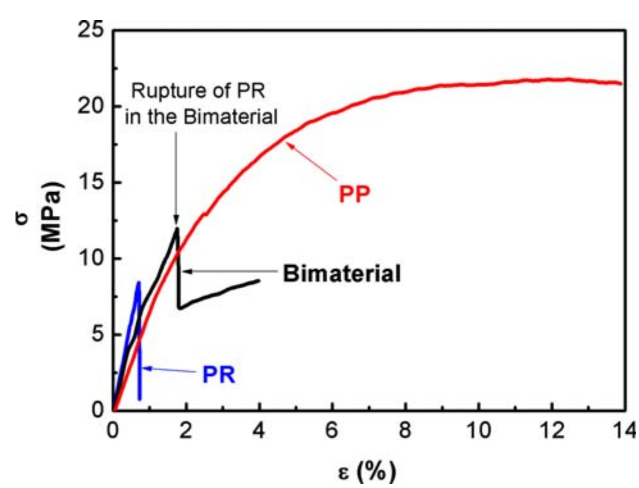


Fig. 6 Tensile stress-strain curve of a PR/PP bimaterial. Graphs of neat PR and PP are also shown

Table 2 Elastic modulus of PR calculated using the proposed model and % of difference with the average elastic modulus obtained from direct tensile testing

Specimen #	E_{PR}, MPa	Difference, %
1	1161.1	−7.7
2	1331.9	+5.9
3	1290.9	+2.6
4	1380.4	+9.7
5	1285.0	+2.1

specimens (1258.3 MPa), taken as reference. The maximum difference observed between the modulus predicted by the bimaterial model, Eq 15, and the one determined from direct testing of the PR was 9.7% and occurred for one specimen only. The next larger difference was an underprediction of the elastic modulus by 7.7%, cf. Table 2. Differences under 6% were observed for the three remaining specimens. The average modulus of the PR predicted in Table 2 is 1289.9 MPa, which is within 3% of the experimentally measured value. It is worthwhile to point out that predictions from simple rule of mixtures yield an average modulus of the PR of 1148.1 MPa, which underpredicts the experimentally measured value by ≈9%.

4. Conclusions

A bimaterial model based on laminated beam theory reduced to 1D has been proposed for analysis and determination of the unknown elastic modulus of one of the materials in a bimaterial. Besides the dimensions and modulus of one material, the model requires solely tensile testing of the bimaterial. A parametric analysis was conducted to study the most important variables involved in the bimaterial model. It was found that bimaterials with constituent materials of similar dimensions are not overly sensitivity to small changes in the material and geometric parameters of the bimaterial. On the other hand, bimaterial systems where one of the materials is substantially thicker than the other exhibit high sensitivity to small variations in the bimaterial elastic response, thicknesses, and modulus of the thick material (substrate). The model was validated using a polyester/polypropylene bimaterial and tensile testing of the individual constituent materials, showing a fairly good accuracy. The proposed approach finds direct application, for example, in the determination of thin film elastic modulus in thin film/substrate systems, which will be subject of further investigation.

Acknowledgment

The authors wish to acknowledge the assistance of Paulo Várguez in this work.

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